

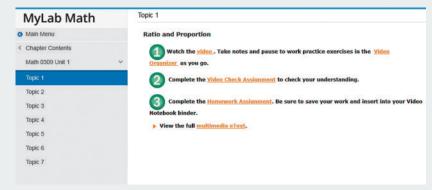


# Get the Most Out of MyLab Math

When it comes to developmental math, we know one size does not fit all. Pearson's solutions offer market-leading content written by our author-educators, tightly integrated with the #1 choice in digital learning-MyLab Math. MyLab Math is the teaching and learning platform that empowers instructors to reach every student. By combining trusted author content with digital tools and a flexible platform, MyLab Math personalizes the learning experience and improves results for each student.

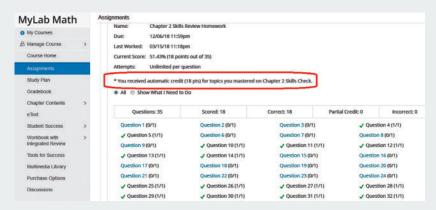
#### • Flexible Platform—

Your course is unique. Whether you'd like to build your own assignments, structure students' work with a learning path, or set prerequisites, you have the flexibility to easily create your course to fit your needs.



- Personalized Learning—Each student learns at a different pace. Personalized learning pinpoints the areas each student needs to practice, giving every student the support they need—when and where they need it—to be successful. A variety of options are available to personalize learning in MyLab Math:
  - With Personalized

    Homework, students take
    a quiz or test and receive
    a subsequent homework
    assignment that is
    personalized based on their
    performance. This way,
    students can focus on just
    the topics they have not yet
    mastered.



• **Skill Builder** offers adaptive practice that is designed to increase students' ability to complete their assignments. By monitoring student performance on their homework, Skill Builder adapts to each student's needs and provides just-in-time, in-assignment practice to help them improve their proficiency of key learning objectives. Available for select MyLab™ courses.

EDITION 9

# Algebra for College Students

Margaret L. Lial

American River College

John Hornsby University of New Orleans

**Terry McGinnis** 



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Cover Image: Borchee/E+/Getty Images

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#### Library of Congress Cataloging-in-Publication Data

Names: Lial, Margaret L., author. | Hornsby, John, 1949- author. | McGinnis,

Terry, author.

Title: Algebra for college students.

Description: 9th edition / Margaret L. Lial (American River College), John Hornsby (University of New Orleans), Terry McGinnis. | Boston: Pearson,

[2020] | Includes index.

Identifiers: LCCN 2019000106 | ISBN 9780135160664 (student edition) | ISBN

0135160669 (student edition) Subjects: LCSH: Algebra--Textbooks.

Classification: LCC QA154.3 .L53 2020 | DDC 512.9--dc23 LC record available at https://lccn.loc.gov/2019000106



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#### **WELCOME TO THE 9TH EDITION**

The first edition of Marge Lial's *Algebra for College Students* was published in 1988, and now we are pleased to present the 9th edition—with the same successful, well-rounded framework that was established over 30 years ago and updated to meet the needs of today's students and professors. The names Lial and Miller, two faculty members from American River College in Sacramento, California, have become synonymous with excellence in Developmental Mathematics, Precalculus, Finite Mathematics, and Applications-Based Calculus.

With Chuck Miller's passing, Marge Lial was joined by a team of carefully selected coauthors who partnered with her. John Hornsby (University of New Orleans) joined Marge in this capacity in 1992, and in 1999, Terry McGinnis became part of this developmental author team. Since Marge's passing in 2012, John and Terry have dedicated themselves to carrying on the Lial/Miller legacy.

In the preface to the first edition of *Intermediate Algebra*, Marge Lial wrote

"... the strongest theme... is a combination of readability and suitability for the book's intended audience: students who are not completely self-confident in mathematics as they come to the course, but who must be self-confident and proficient... by the end of the course."

Today's Lial author team upholds these same standards. With the publication of the 9th edition of *Algebra for College Students*, we proudly present a complete course program for students who need developmental algebra. Revisions to the core text, working in concert with such innovations in the MyLab Math course as Skill Builder and Learning Catalytics, combine to provide superior learning opportunities appropriate for all types of courses (traditional, hybrid, online).

We hope you enjoy using it as much as we have enjoyed writing it. We welcome any feedback that you have as you review and use this text.

#### WHAT'S NEW IN THIS EDITION?

We are pleased to offer the following new features and resources in the text and MyLab.

**IMPROVED STUDY SKILLS** These special activities are now grouped together at the front of the text, prior to Chapter R. **Study Skills Reminders** that refer students to specific Study Skills are found liberally throughout the text. Many Study Skills

now include a *Now Try This* section to help students implement the specific skill.

**REVISED EXPOSITION** With each edition of the text, we continue to polish and improve discussions and presentations of topics to increase readability and student understanding. This edition is no exception.

**NEW FIGURES AND DIAGRAMS** For visual learners, we have included more than 50 new mathematical figures, graphs, and diagrams, including several new "hand drawn" style graphs. These are meant to suggest what a student who is graphing with paper and pencil should obtain. We use this style when introducing a particular type of graph for the first time.

**ENHANCED USE OF PEDAGOGICAL COLOR** We have thoroughly reviewed the use of pedagogical color in discussions and examples and have increased its use whenever doing so would enhance concept development, emphasize important steps, or highlight key procedures.

**INCREASED Concept Check AND WHAT WENT WRONG? EXERCISES** The number of **Concept Check** exercises, which facilitate students' mathematical thinking and conceptual understanding, and which begin each exercise set, has been increased. We have also more than doubled the number of **WHAT WENT WRONG?** exercises that highlight common student errors.

INCREASED RELATING CONCEPTS EXERCISES We have doubled the number of these flexible groups of exercises, which are located at the end of many exercise sets. These sets of problems were specifically written to help students tie concepts together, compare and contrast ideas, identify and describe patterns, and extend concepts to new situations. They may be used by individual students or by pairs or small groups working collaboratively. All answers to these exercises appear in the student answer section.

ENHANCED MYLAB MATH RESOURCES MyLab exercise coverage in the revision has been expanded, and video coverage has also been expanded and updated to a modern format for today's students. WHAT WENT WRONG? problems and all RELATING CONCEPTS exercise sets (both even- and odd-numbered problems) are now assignable in MyLab Math.

**SKILL BUILDER** These exercises offer just-in-time additional adaptive practice in MyLab Math. The adaptive engine tracks student performance and delivers, to each individual, questions that adapt to his or her level of understanding. This new feature enables instructors to assign fewer questions for

homework, allowing students to complete as many or as few questions as they need.

**LEARNING CATALYTICS** This new student response tool uses students' own devices to engage them in the learning process. Problems that draw on prerequisite skills are included at the beginning of each section to gauge student readiness for the section. Accessible through MyLab Math and customizable to instructors' specific needs, these problems can be used to generate class discussion, promote peer-to-peer learning, and provide real-time feedback to instructors. More information can be found via the Learning Catalytics link in MyLab Math. Specific exercises notated in the text can be found by searching LialACS# where the # is the chapter number.

#### **CONTENT CHANGES**

Specific content changes include the following:

- Exercise sets have been scrutinized and updated with a renewed focus on conceptual understanding and skill development. Even and odd pairing of the exercises, an important feature of the text, has been carefully reviewed.
- **Real world data** in all examples and exercises and their accompanying graphs has been updated.
- An increased emphasis on fractions, decimals, and percents appears throughout the text. Chapter R begins with a new section that thoroughly reviews these topics. And we have included an all-new set of Cumulative Review Exercises, many of which focus on fractions, decimals, and percents, at the end of Chapter 1. Sets of Cumulative Review Exercises in subsequent chapters now begin with new exercises that review skills related to these topics.
- Solution sets of linear inequalities in Sections 1.5–1.7 are now graphed first before writing them using interval notation.
- Scientific notation is covered in a separate section in Chapter 4.
- Presentations of the following topics have been enhanced and expanded, often including new examples and exercises:

Evaluating exponential expressions (Section R.4)

Geometric interpretation of slope as rise/run (Section 2.2)

Identifying functions and domains from equations (Section 2.6)

Solving systems of linear equations in three variables (Section 3.2)

Determining strategies for factoring polynomials (Section 5.4)

Solving quadratic equations with double solutions (Section 5.5)

Solving rational equations (Section 6.4)

Concepts and relationships among real numbers, nonreal complex numbers, and imaginary numbers; simplifying powers of i (Section 7.7)

Solving quadratic equations using the quadratic formula (Section 8.2)

Testing for symmetry with respect to an axis or the origin (Section 9.4)

Solving exponential and logarithmic equations (Sections 10.2, 10.3)

Graphing polynomial and rational functions (Sections 11.3, 11.4)

Graphing systems of linear inequalities (Section 12.4)

#### LIAL DEVELOPMENTAL HALLMARK FEATURES

We have enhanced the following popular features, each of which is designed to increase ease of use by students and/ or instructors.

- Emphasis on Problem-Solving We introduce our sixstep problem-solving method in Chapter 2 and integrate it throughout the text. The six steps, Read, Assign a Variable, Write an Equation, Solve, State the Answer, and Check, are emphasized in boldface type and repeated in examples and exercises to reinforce the problem-solving process for students. We also provide students with PROBLEM-SOLVING HINT boxes that feature helpful problem-solving tips and strategies.
- Helpful Learning Objectives We begin each section with clearly stated, numbered objectives, and the included material is directly keyed to these objectives so that students and instructors know exactly what is covered in each section.
- Cautions and Notes One of the most popular features of previous editions is our inclusion of information marked
   CAUTION and NOTE to warn students about common errors and to emphasize important ideas throughout the exposition. The updated text design makes them easy to spot.
- Comprehensive Examples The new edition features a multitude of step-by-step, worked-out examples that include pedagogical color, helpful side comments, and special pointers. We give special attention to checking example solutions—more checks, designated using a special CHECK tag and ✓, are included than in past editions.

- More Pointers There are more pointers in examples and discussions throughout this edition of the text. They provide students with important on-the-spot reminders, as well as warnings about common pitfalls.
- Numerous Now Try Problems These margin exercises, with answers immediately available at the bottom of the page, have been carefully written to correspond to every example in the text. This key feature allows students to immediately practice the material in preparation for the exercise sets.
- Updated Figures, Photos, and Hand-Drawn Graphs
   Today's students are more visually oriented than ever.
   As a result, we provide detailed mathematical figures, diagrams, tables, and graphs, including a "hand-drawn" style of graphs, whenever possible. We have incorporated depictions of well-known mathematicians, as well as appealing photos to accompany applications in examples and exercises.
- *Relevant Real-Life Applications* We include many new or updated applications from fields such as business, pop culture, sports, technology, and the health sciences that show the relevance of algebra to daily life.
- Extensive and Varied Exercise Sets The text contains a wealth of exercises to provide students with opportunities to practice, apply, connect, review, and extend the skills they are learning. Numerous illustrations, tables, graphs, and photos help students visualize the problems they are solving. Problem types include skill building and writing exercises, as well as applications, matching, true/false, multiple-choice, and fill-in-the-blank problems. Special types of exercises include Concept Check, WHAT WENT WRONG?, Extending Skills, and RELATING CONCEPTS.
- Special Summary Exercises We include a set of these popular in-chapter exercises in every chapter. They provide students with the all-important mixed review problems they need to master topics and often include summaries of solution methods and/or additional examples.
- *Extensive Review Opportunities* We conclude each chapter with the following review components:

A Chapter Summary that features a helpful list of Key Terms organized by section, New Symbols, a Test Your Word Power vocabulary quiz (with answers immediately following), and a Quick Review of each section's main concepts, complete with additional examples.

A comprehensive set of **Chapter Review Exercises**, keyed to individual sections for easy student reference.

A set of **Mixed Review Exercises** that helps students further synthesize concepts and skills.

A **Chapter Test** that students can take under test conditions to see how well they have mastered the chapter material.

A set of **Cumulative Review Exercises** for ongoing review that covers material going back to Chapter R.

• *Comprehensive Glossary* The online Glossary includes key terms and definitions (with section references) from throughout the text.

#### **ACKNOWLEDGMENTS**

The comments, criticisms, and suggestions of users, nonusers, instructors, and students have positively shaped this text over the years, and we are most grateful for the many responses we have received. The feedback gathered for this edition was particularly helpful.

We especially wish to thank the following individuals who provided invaluable suggestions.

Barbara Aaker, Community College of Denver Kim Bennekin, Georgia Perimeter College Dixie Blackinton, Weber State University Eun Cha, College of Southern Nevada, Charleston Callie Daniels, St. Charles Community College Cheryl Davids, Central Carolina Technical College Robert Diaz, Fullerton College Chris Diorietes, Fayetteville Technical Community College Sylvia Dreyfus, Meridian Community College Sabine Eggleston, Edison State College LaTonya Ellis, Bishop State Community College Beverly Hall, Fayetteville Technical Community College Loretta Hart, NHTI, Concord's Community College Sandee House, Georgia Perimeter College Joe Howe, St. Charles Community College Lynette King, Gadsden State Community College Linda Kodama, Windward Community College Carlea McAvoy, South Puget Sound Community College James Metz, Kapi 'olani Community College Jean Millen, Georgia Perimeter College Molly Misko, Gadsden State Community College Charles Patterson, Louisiana Tech Jane Roads, Moberly Area Community College Melanie Smith, Bishop State Community College Erik Stubsten, Chattanooga State Technical Community

Tong Wagner, *Greenville Technical College*Rick Woodmansee, *Sacramento City College*Sessia Wyche, *University of Texas at Brownsville* 

Over the years, we have come to rely on an extensive team of experienced professionals. Our sincere thanks go to these dedicated individuals at Pearson who worked long and hard to make this revision a success. We would like to thank Michael Hirsch, Matthew Summers, Karen Montgomery, Alicia Frankel, Lauren Morse, Vicki Dreyfus, Stacey Miller, Noelle Saligumba, Eric Gregg, and all of the Pearson math team for helping with the revision of the text.

We are especially pleased to welcome Callie Daniels, who has taught from our texts for many years, to our team. Her assistance has been invaluable. She thoroughly reviewed all chapters and helped extensively with manuscript preparation.

We are grateful to Carol Merrigan for her excellent production work. We appreciate her positive attitude, responsiveness, and expert skills. We would also like to thank Pearson CSC for their production work; Emily Keaton for her detailed help in updating real data applications; Connie

Day for supplying her copyediting expertise; Pearson CSC for their photo research; and Lucie Haskins for producing another accurate, useful index. Paul Lorczak and Hal Whipple did a thorough, timely job accuracy-checking the page proofs and answers, and Sarah Sponholz checked the index.

We particularly thank the many students and instructors who have used this text over the years. You are the reason we do what we do. It is our hope that we have positively impacted your mathematics journey. We would welcome any comments or suggestions you might have via email to math@pearson.com.

John Hornsby Terry McGinnis

#### **DEDICATION**

To Wayne and Sandra

E.J.H.

To Andrew and Tyler

Mom



# **Resources for Success**

# Get the Most Out of MyLab Math for *Algebra for College Students*, Ninth Edition by Lial, Hornsby, McGinnis

The Lial team has helped thousands of students learn algebra with an approachable, teacherly writing style and balance of skill and concept development. With this revision, the series retains the hallmarks that have helped students succeed in math, and includes new and updated digital tools in the MyLab Math course.

Take advantage of the following resources to get the most out of your MyLab Math course.

#### **Get Students Prepared with Integrated Review**

Every student enters class with different levels of preparedness and prerequisite knowledge. To ensure students are caught up on prior skills, every Lial MyLab course now includes Integrated Review.

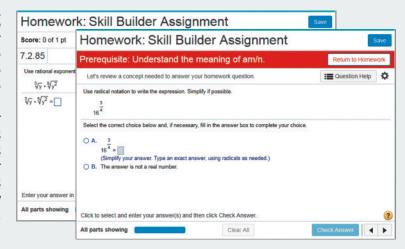


New! Integrated Review provides embedded and personalized review of prerequisite topics within relevant chapters. Students can check their prerequisite skills, and receive personalized practice on the topics they need to focus on, with study aids like worksheets and videos also available to help.

Integrated Review assignments are premade and available to assign in the Assignment Manager.

#### **Personalize Learning**

New! Skill Builder exercises offer just-in-time additional adaptive practice. The adaptive engine tracks student performance and delivers questions to each individual that adapt to his or her level of understanding. This new feature allows instructors to assign fewer questions for homework, allowing students to complete as many or as few questions as they need.





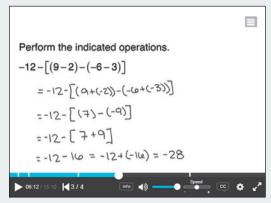
# **Resources for Success**

#### **Support Students Whenever, Wherever**

**Updated!** The **complete video program** for the Lial series includes:

- Full Section Lecture Videos
- Solution clips for select exercises
- Chapter Test Prep videos
- Short Quick Review videos that recap each section

Full Section Lecture Videos are also available as shorter, objective-level videos. No matter your students' needs—if they missed class, need help solving a problem, or want a short summary of a section's concepts—they can get support whenever they need it, wherever they need it. Much of the video series has been updated in a modern presentation format.





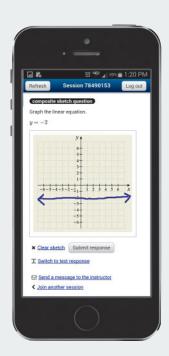
#### **Foster a Growth Mindset**

New! A Mindset module is available in the course, with mindset-focused videos and exercises that encourage students to maintain a positive attitude about learning, value their own ability to grow, and view mistakes as a learning opportunity.

#### **Get Students Engaged**

New! Learning Catalytics Learning Catalytics is an interactive student response tool that uses students' smartphones, tablets, or laptops to engage them in more sophisticated tasks and thinking.

In addition to a library of developmental math questions, Learning Catalytics questions created specifically for this text are pre-built to make it easy for instructors to begin using this tool! These questions, which cover prerequisite skills before each section, are noted in the margin of the Annotated Instructor's Edition, and can be found in Learning Catalytics by searching for "LialACS#", where # is the chapter number.



# Pearson MyLab

# **Resources for Success**

#### **Instructor Resources**

#### **Annotated Instructor's Edition**

Contains all the content found in the student edition, plus answers to even and odd exercises on the same text page, and Teaching Tips and Classroom Examples throughout the text placed at key points.

The resources below are available through Pearson's Instructor Resource Center, or from MyLab Math.

# Instructor's Resource Manual with Tests

Includes mini-lectures for each text section, several forms of tests per chapter—two diagnostic pretests, four free-response and two multiple-choice test forms per chapter, and two final exams.

#### Instructor's Solutions Manual

Contains detailed, worked-out solutions to all exercises in the text.

#### TestGen<sup>®</sup>

Enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions.

#### **PowerPoint Lecture Slides**

Available for download only, these slides present key concepts and definitions from the text. Accessible versions of the PowerPoint slides are also available for students who are visionimpaired.

#### **Student Resources**

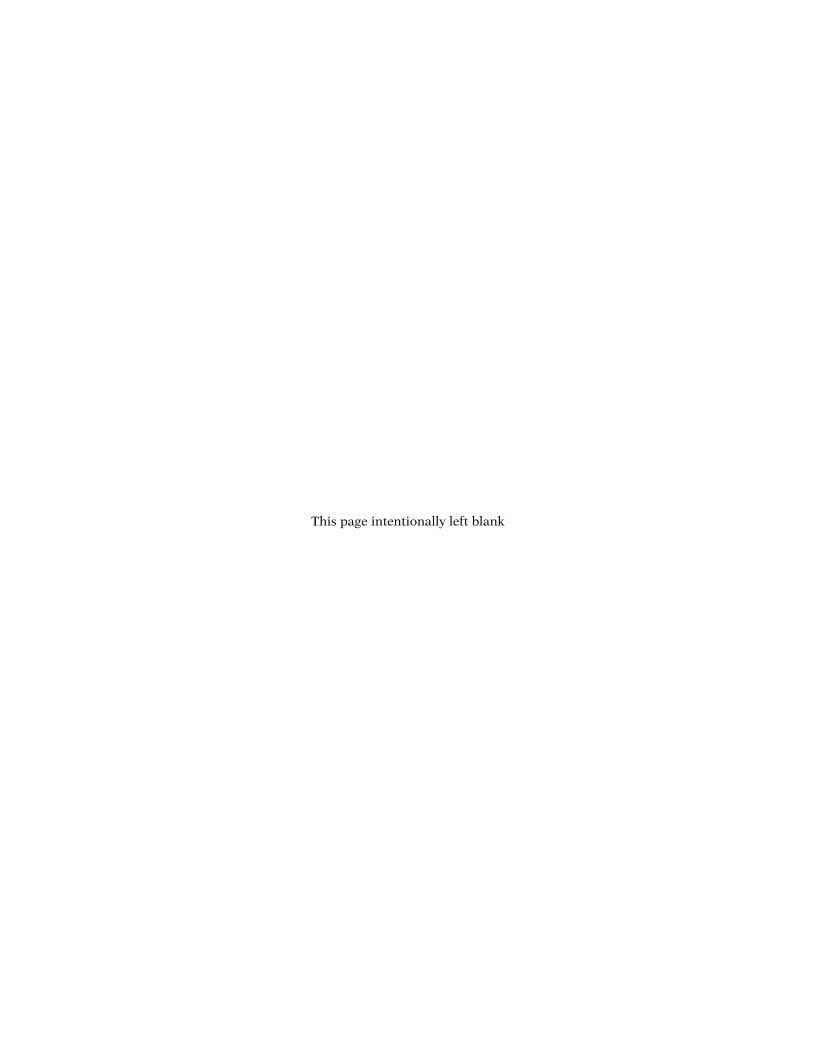
#### **Guided Notebook**

This Guided Notebook helps students keep their work organized as they work through their course. The notebook includes:

- Guided Examples that are worked out for students, plus corresponding Now Try This exercises for each text objective.
- Extra practice exercises for every section of the text, with ample space for students to show their work
- Learning objectives and key vocabulary terms for every text section, along with vocabulary practice problems.

#### **Student Solutions Manual**

Provides completely worked-out solutions to the odd-numbered section exercises and to all exercises in the Now Trys, Relating Concepts, Chapter Reviews, Mixed Reviews, Chapter Tests, and Cumulative Reviews. Available at no additional charge in the MyLab Math course.



# **Using Your Math Text**

Your text is a valuable resource. You will learn more if you make full use of the features it offers.

#### *Now* **TRY THIS**

#### **General Features of This Text**

Locate each feature, and complete any blanks.

- Table of Contents This is located at the front of the text.

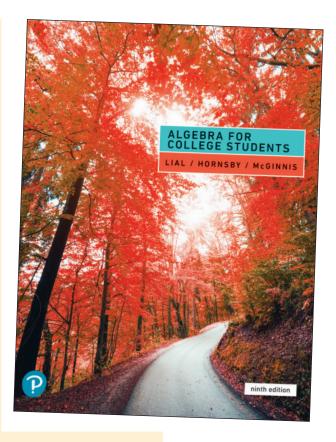
  Find it and mark the chapters and sections you will cover, as noted on your course syllabus.
- Answer Section This is located at the back of the text.
   Tab this section so you can easily refer to it when doing homework or reviewing for tests.
- List of Formulas This helpful list of geometric formulas, along with review information on triangles and angles, is found at the back of the text.

The formula for the volume of a cube is \_\_\_\_\_\_.

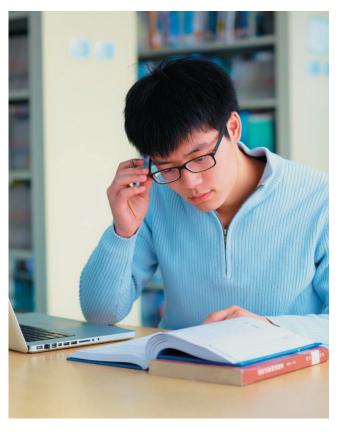
#### **Specific Features of This Text**

Look through Chapter 1 and give the number of a page that includes an example of each of the following specific features.

 Objectives The objectives are listed at the beginning of each section and again within the section as the corresponding material is presented. Once you finish a section, ask yourself if you have accomplished them.
 See page \_\_\_\_\_\_\_.



- Vocabulary List Important vocabulary is listed at the beginning of each section. You should be able to define these terms when you finish a section. See page \_\_\_\_\_\_.
- Now Try Exercises These margin exercises allow you to immediately practice the
  material covered in the examples and prepare you for the exercises. Check your results
  using the answers at the bottom of the page. See page \_\_\_\_\_\_.
- **Pointers** These small, shaded balloons provide on-the-spot warnings and reminders, point out key steps, and give other helpful tips. **See page** \_\_\_\_\_\_.
- Cautions These provide warnings about common errors that students often make or trouble spots to avoid. See page \_\_\_\_\_\_.
- **Notes** These provide additional explanations or emphasize other important ideas. **See page** \_\_\_\_\_\_.
- Problem-Solving Hints These boxes give helpful tips or strategies to use when you
  work applications. See page \_\_\_\_\_\_.



# **Reading Your Math Text**

Take time to read each section and its examples before doing your homework. You will learn more and be better prepared to work the exercises your instructor assigns.

#### Approaches to Reading Your Math Text

**Student A** learns best by listening to her teacher explain things. She "gets it" when she sees the instructor work problems. She previews the section before the lecture, so she knows generally what to expect. **Student A carefully reads the section in her text** *AFTER* **she hears the classroom lecture on the topic.** 

**Student B** learns best by reading on his own. He reads the section and works through the examples before coming to class. That way, he knows what the teacher is going to talk about and what questions he wants to ask. **Student B carefully reads the section in his text BEFORE** he hears the classroom lecture on the topic.

Which of these reading approaches works best for you—that of Student A or Student B?

#### **Tips for Reading Your Math Text**

- Turn off your cell phone and the TV. You will be able to concentrate more fully on what you are reading.
- Survey the material. Glance over the assigned material to get an idea of the "big picture."
   Look at the list of objectives to see what you will be learning.
- Read slowly. Read only one section—or even part of a section—at a sitting, with paper and pencil in hand.
- Pay special attention to important information given in colored boxes or set in boldface type. Highlight any additional information you find helpful.
- Study the examples carefully. Pay particular attention to the blue side comments and any pointer balloons.
- Do the Now Try exercises in the margin on separate paper as you go. These problems
  mirror the examples and prepare you for the exercise set. Check your answers with those
  given at the bottom of the page.
- Make study cards as you read. Make cards for new vocabulary, rules, procedures, formulas, and sample problems.
- Mark anything you don't understand. ASK QUESTIONS in class—everyone will benefit.
   Follow up with your instructor, as needed.

#### **Now TRY THIS**

Think through and answer each question.

- 1. Which two or three reading tips given above will you try this week?
- **2.** Did the tips you selected improve your ability to read and understand the material? Explain.

# **Taking Lecture Notes**

#### Come to class prepared.

- Bring paper, pencils, notebook, text, completed homework, and any other materials you need.
- Arrive 10–15 minutes early if possible. Use the time before class to review your notes or study cards from the last class period.
- Select a seat carefully so that you can hear and see what is going on.

# Study the set of sample math notes given at the right.

- Use a new page for each day's lecture.
- Include the date and title of the day's lecture topic.
- Skip lines and write neatly to make reading easier.
- Include cautions and warnings to emphasize common errors to avoid.
- Mark important concepts with stars, underlining, etc.
- Use two columns, which allows an example and its explanation to be close together.
- Use brackets and arrows to clearly show steps, related material, etc.
- Highlight any material and/or information that your instructor emphasizes.
   Instructors often give "clues" about material that will definitely be on an exam.

Translating Words to Express and Equations	ions	Sept. 1
Problem solving: key words or phrases tran expressions.	ıslate to alge	braic
Caution Subtraction is not commutative the order does matter.	e;	
	Correct	Wrong
Examples: 10 less than a number	x -10	10-x
a number subtracted from 10	10 - x	x -10
10 minus a number	10 - x	x -10
A phrase (part of a sentence)	A sente	rnce
algebraic expression	equation w	14.
Note difference  No equality symbol in an exp Equation has an equality sym  Pay close attention to exact wording of the	ubol. 3x	+ 2 + 2 = 14
watch for commas.  The quotient of (a number) and (the	number plu.	s 4) is 28.
The quotient of (a number) and (4), p	$\frac{x}{x+4}$	= 28
$\frac{x}{4}$	+ x mmas separat	28 e
this fr	rom division p	part

Consider using a three-ring binder to organize your notes, class handouts, and completed homework.

#### *Now* **TRY THIS**

With a study partner or in a small group, compare lecture notes. Then answer each question.

- 1. What are you doing to show main points in your notes (such as boxing, using stars, etc.)?
- 2. In what ways do you set off explanations from worked problems and subpoints (such as indenting, using arrows, circling, etc.)?
- **3.** What new ideas did you learn by examining your classmates' notes?
- **4.** What new techniques will you try when taking notes in future lectures?

# STUDY SKILL 4



## **Completing Your Homework**

You are ready to do your homework **AFTER** you have read the corresponding text section and worked through the examples and Now Try exercises.

#### **Homework Tips**

- Keep distractions and potential interruptions to a minimum. Turn off your cell phone and the TV. Find a quiet, comfortable place to work, away from a lot of other people, so you can concentrate on what you are doing.
- Review your class notes. Pay particular attention to anything your instructor emphasized during the lecture on this material.
- Survey the exercise set. Glance over the problems that your instructor has assigned to get a general idea of the types of exercises you will be working. Skim directions, and note any references to section examples.
- Work problems neatly. NEVER do your math homework in pen. Use pencil and write legibly, so others can read your work. Skip lines between steps. Clearly separate problems from each other.
- Show all your work. It is tempting to take shortcuts. Include ALL steps.
- Check your work frequently to make sure you are on the right track. It is hard to unlearn a mistake. For all odd-numbered problems, answers are given in the back of the text.
- If you have trouble with a problem, refer to the corresponding worked example in the section. The exercise directions will often reference specific examples to review. Pay attention to every line of the worked example to see how to get from step to step.
- If you have trouble with an even-numbered problem, work the corresponding oddnumbered problem. Check your answer in the back of the text, and apply the same steps to work the even-numbered problem.
- If you have genuinely tried to work a problem but have not been able to complete it in a *reasonable* amount of time, it's ok to STOP. Mark these problems. Ask for help at your school's tutor center or from fellow classmates, study partners, or your instructor.
- Do some homework problems every day. This is a good habit, even if your math class does not meet each day.

#### *Now* **TRY THIS**

#### Think through and answer each question.

- 1. What is your instructor's policy regarding homework?
- 2. Think about your current approach to doing homework. Be honest in your assessment.
  - (a) What are you doing that is working well?
  - (b) What improvements could you make?
- 3. Which one or two homework tips will you try this week?
- 4. In the event that you need help with homework, what resources are available? When does your instructor hold office hours?

# **Using Study Cards**

You may have used "flash cards" in other classes. In math, "study cards" can help you remember terms and definitions, procedures, and concepts. Use study cards to

- Help you understand and learn the material;
- Quickly review when you have a few minutes;
- Review before a guiz or test.

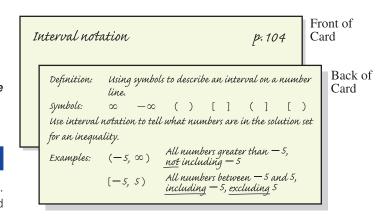
One of the advantages of study cards is that you learn the material while you are making them.

#### **Vocabulary Cards**

Put the word and a page reference on the front of the card. On the back, write the definition, an example, any related words, and a sample problem (if appropriate).

#### Procedure ("Steps") Cards

Write the name of the procedure on the front of the card. Then write each step in words. On the back of the card, put an example showing each step.



#### Solving a Linear Inequality

p.108

Front of

Card

1. Simplify each side separately. (Clear parentheses, fractions, and decimals and combine like terms.)

- 2. Isolate variable terms on one side. (Add or subtract the same number from each side.)
- 3. Isolate the variable. (Divide each side by the same number; if dividing by a negative number, reverse direction of inequality.)

Back of Solve  $-3(x+4)+2 \ge 7-x$  and graph the solution set. Card Clear parentheses. Combine like terms. Both sides are simplified. Variable term still not isolated. Add 10 to each side. Divide each side by -2; dividing by negative, reverse direction of inequality symbol.

# $-3(x+4)+2 \ge 7-x$ $-3x - 12 + 2 \ge 7 - x$ $-3x - 10 \ge 7 - x$ $-3x-10+x \ge 7-x+x$ Add x to each side. $-2x - 10 \ge 7$ $-2x - 10 + 10 \ge 7 + 10$

#### **Practice Problem Cards**

Write a problem with direction words (like solve, simplify) on the front of the card, and work the problem on the back. Make one for each type of problem you learn.

#### **Now TRY THIS**

Make a vocabulary card, a procedure card, and a practice problem card for material that you are learning or reviewing.

Solve this inequality. Give the solution set in both interval and graph forms.

-.3

$$-5x - 4 \ge 11$$

-5x - 4 > 11Neither side can be simplified.  $-5x - 4 + 4 \ge 11 + 4$ Add 4 to each side -5x≥15 Divide each side by -5. Reverse direction of inequality symbol <u>−5x(\$)15</u> because dividing by <u>negative</u> number.  $x \leq -3$ Solution set in interval form;  $(-\infty, -3]$ all numbers less than or equal to −3, including –3. Graph of solution set.

Back of Card

Front of

Card

# **Managing Your Time**

Many college students juggle a busy schedule and multiple responsibilities, including school, work, and family demands.



#### **Time Management Tips**

- Read the syllabus for each class. Understand class policies, such as attendance, late homework, and make-up tests. Find out how you are graded.
- Make a semester or quarter calendar. Put test dates and major due dates for all your classes on the same calendar. Try using a different color for each class.
- Make a weekly schedule. After you fill in your classes and other regular responsibilities, block off some study periods. Aim for 2 hours of study for each 1 hour in class.
- Choose a regular study time and place (such as the campus library). Routine helps.
- Keep distractions to a minimum. Get the most out of the time you have set aside for studying by limiting interruptions. Turn off your cell phone. Take a break from social media. Avoid studying in front of the TV.
- Make "to-do" lists. Number tasks in order of importance. To see your progress, cross off tasks as you complete them.
- Break big assignments into smaller chunks. Don't wait until the last minute to begin big
  assignments or to study for tests. Make deadlines for each smaller chunk so that you stay
  on schedule.
- Take breaks when studying. Do not try to study for hours at a time. Take a 10-minute break each hour or so.
- Ask for help when you need it. Talk with your instructor during office hours. Make use
  of the learning/tutoring center, counseling office, or other resources available at your
  school.

#### *Now* **TRY THIS**

#### Work through the following, answering any questions.

- 1. Evaluate when and where you are currently studying. Are these places quiet and comfortable? Are you studying when you are most alert?
- 2. Which of the above tips will you try this week to improve your time management?
- **3.** Create a weekly calendar that includes your class times, study times, and other family and/or work obligations.
- **4.** Once the week is over, evaluate how these tips worked. Did you use your calendar and stick to it? What will you do differently next week?
- **5.** Ask classmates, friends, and/or family members for tips on how they manage their time. Try any that you think might work for you.

## Reviewing a Chapter

Your text provides extensive material to help you prepare for quizzes or tests in this course. Refer to the **Chapter 1 Summary** as you read through the following techniques.

#### Techniques for Reviewing a Chapter

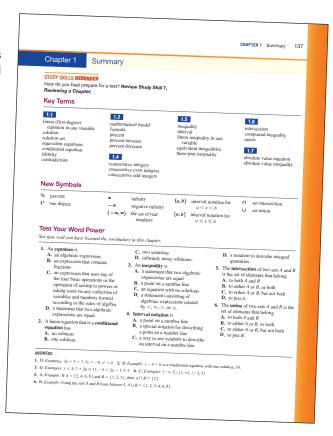
- Review the Key Terms and any New Symbols. Make a study card for each. Include a definition, an example, a sketch (if appropriate), and a section or page reference.
- Take the Test Your Word Power quiz to check your understanding of new vocabulary. The answers immediately follow.
- Read the Quick Review. Pay special attention to the headings.
   Study the explanations and examples given for each concept. Try to think about the whole chapter.
- Reread your lecture notes. Focus on what your instructor has emphasized in class, and review that material in your text.
- Look over your homework. Pay special attention to any trouble spots.
- Work the Review Exercises. They are grouped by section.
   Answers are included at the back of the text.
  - Pay attention to direction words, such as simplify, solve, and evaluate.
  - ► Are your answers exact and complete? Did you include the correct labels, such as \$, cm², ft, etc.?
  - Make study cards for difficult problems.
- Work the Mixed Review Exercises. They are in random order. Check your answers in the answer section at the back of the text.
- Take the Chapter Test under test conditions.
  - ▶ Time yourself.
  - Use a calculator or notes only if your instructor permits them on tests.
  - Take the test in one sitting.
  - Show all your work.
  - Check your answers in the answer section. Section references are provided.

**Reviewing a chapter takes time.** Avoid rushing through your review in one night. Use the suggestions over a few days or evenings to better understand and remember the material.

#### **Now TRY THIS**

Follow these reviewing techniques to prepare for your next test. Then answer each question.

- 1. How much time did you spend reviewing for your test? Was it enough?
- 2. Which reviewing techniques worked best for you?
- **3.** Are you investing enough time and effort to really *know* the material and set yourself up for success? Explain.
- 4. What will you do differently when reviewing for your next test?



# **Taking Math Tests**

Techniques to Improve Your Test Score	Comments
Come prepared with a pencil, eraser, paper, and calculator, if allowed.	Working in pencil lets you erase, keeping your work neat.
Scan the entire test, note the point values of different problems, and plan your time accordingly.	To do 20 problems in 50 minutes, allow $50 \div 20 = 2.5$ minutes per problem. Spend less time on easier problems.
Do a "knowledge dump" when you get the test. Write important notes, such as formulas, in a corner of the test for reference.	Writing down tips and other special information that you've learned at the beginning allows you to relax as you take the test.
Read directions carefully, and circle any significant words. When you finish a problem, reread the directions. Did you do what was asked?	Pay attention to any announcements written on the board or made by your instructor. Ask if you don't understand something.
<b>Show all your work.</b> Many teachers give partial credit if some steps are correct, even if the final answer is wrong. Write neatly.	If your teacher can't read your writing, you won't get credit for it. If you need more space to work, ask to use extra paper.
Write down anything that might help solve a problem: a formula, a diagram, etc. If necessary, circle the problem and come back to it later. Do not erase anything you wrote down.	If you know even a little bit about a problem, write it down. The answer may come to you as you work on it, or you may get partial credit. Don't spend too long on any one problem.
If you can't solve a problem, make a guess. Do not change it unless you find an obvious mistake.	Have a good reason for changing an answer. Your first guess is usually your best bet.
Check that the answer to an application problem is reasonable and makes sense. Reread the problem to make sure you've answered the question.	Use common sense. Can the father really be seven years old? Would a month's rent be \$32,140? Remember to label your answer if needed: \$, years, inches, etc.
Check for careless errors. Rework each problem without looking at your previous work. Then compare the two answers.	Reworking a problem from the begin- ning forces you to rethink it. If possible, use a different method to solve the problem.



#### *Now* **TRY THIS**

#### Think through and answer each question.

- 1. What two or three tips will you try when you take your next math test?
- **2.** How did the tips you selected work for you when you took your math test?
- 3. What will you do differently when taking your next math test?
- **4.** Ask several classmates how they prepare for math tests. Did you learn any new preparation ideas?

# **Analyzing Your Test Results**

An exam is a learning opportunity—learn from your mistakes. After a test is returned, do the following:

- Note what you got wrong and why you had points deducted.
- Figure out how to solve the problems you missed. Check your text or notes, or ask your instructor. Rework the problems correctly.
- Keep all quizzes and tests that are returned to you. Use them to study for future tests and the final exam.

#### Typical Reasons for Errors on Math Tests

- 1. You read the directions wrong.
- 2. You read the question wrong or skipped over something.
- 3. You made a computation error.
- **4.** You made a careless error. (For example, you incorrectly copied a correct answer onto a separate answer sheet.)
- 5. Your answer was not complete.
- 6. You labeled your answer wrong. (For example, you labeled an answer "ft" instead of "ft2.")
- 7. You didn't show your work.
- 8. You didn't understand a concept.
- 9. You were unable to set up the problem (in an application).
- 10. You were unable to apply a procedure.

#### These are test-taking errors.

They are easy to correct if you read carefully, show all your work, proofread, and double-check units and labels.

These are test preparation errors. Be sure to practice all the kinds of problems that you will see on tests.

#### **Now TRY THIS**

#### Work through the following, answering any questions.

- Use the sample charts at the right to track your test-taking progress. Refer to the tests you have taken so far in your course. For each test, check the appropriate box in the charts to indicate that you made an error in a particular category.
- 2. What test-taking errors did you make? Do you notice any patterns?
- 3. What test preparation errors did you make? Do you notice any patterns?
- **4.** What will you do to avoid these kinds of errors on your next test?

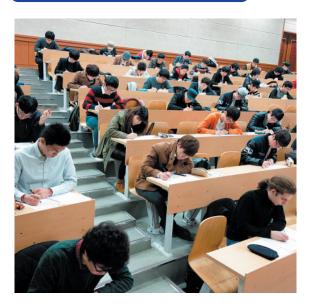
#### Test-Taking Errors

Test	Read directions wrong	Read question wrong	Made computation error	Made careless error	Answer not complete	Answer labeled wrong	Didn't show work
1							
2							
3							

#### Test Preparation Errors

Test	Didn't understand concept	Didn't set up problem correctly	Couldn't apply a procedure
1			
2			
3			

# STUDY SKILL 10



# **Preparing for Your Math Final Exam**

Your math final exam is likely to be a comprehensive exam, which means it will cover material from the entire term. One way to prepare for it now is by working a set of Cumulative Review Exercises each time your class finishes a chapter. This continual review will help you remember concepts and procedures as you progress through the course.

#### **Final Exam Preparation Suggestions**

- Figure out the grade you need to earn on the final exam to get the course grade you want. Check your course syllabus for grading policies, or ask your instructor if you are not sure.
- 2. Create a final exam week plan. Set priorities that allow you to spend extra time studying. This may mean making adjustments, in advance, in your work schedule or enlisting extra help with family responsibilities.
- 3. Use the following suggestions to guide your studying.
  - Begin reviewing several days before the final exam. DON'T wait until the last minute.
  - Know exactly which chapters and sections will be covered on the exam.
  - Divide up the chapters. Decide how much you will review each day.
  - Keep returned quizzes and tests. Use them to review.
  - Practice all types of problems. Use the Cumulative Review Exercises at the end of
    each chapter in your text beginning in Chapter 1. All answers, with section references,
    are given in the answer section at the back of the text.
  - Review or rewrite your notes to create summaries of important information.
  - Make study cards for all types of problems. Carry the cards with you, and review them whenever you have a few minutes.
  - Take plenty of short breaks as you study to reduce physical and mental stress. Exercising, listening to music, and enjoying a favorite activity are effective stress busters.

Finally, DON'T stay up all night the night before an exam—get a good night's sleep.

#### **Now TRY THIS**

#### Think through and answer each question.

- **1.** How many points do you need to earn on your math final exam to get the grade you want in your course?
- 2. What adjustments to your usual routine or schedule do you need to make for final exam week? List two or three.
- **3.** Which of the suggestions for studying will you use as you prepare for your math final exam? List two or three.
- 4. Analyze your final exam results. How will you prepare differently next time?

# REVIEW OF THE REAL NUMBER SYSTEM



- R.1 Fractions, Decimals, and Percents
- R.2 Basic Concepts from Algebra
- **R.3 Operations on Real Numbers**
- R.4 Exponents, Roots, and Order of Operations
- **R.5 Properties of Real Numbers**

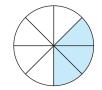
# **R.1**

## Fractions, Decimals, and Percents

#### **OBJECTIVES**

- 1 Write fractions in lowest terms.
- 2 Convert between improper fractions and mixed numbers.
- **3** Perform operations with fractions.
- **4** Write decimals as fractions.
- **5** Perform operations with decimals.
- **6** Write fractions as decimals.
- 7 Write percents as decimals and decimals as percents.
- Write percents as fractions and fractions as percents.

Recall that **fractions** are a way to represent parts of a whole. See **FIGURE 1**. In a fraction, the **numerator** gives the number of parts being represented. The **denominator** gives the total number of equal parts in the whole. The fraction bar represents division  $\left(\frac{a}{b} = a \div b\right)$ .



The shaded region represents  $\frac{3}{8}$  of the circle.

Fraction bar 
$$\rightarrow \frac{3}{8} \leftarrow \text{Numerator}$$
Penominator

A fraction is classified as either a **proper fraction** or an **improper fraction**.

Proper fractions  $\frac{1}{5}$ ,  $\frac{2}{7}$ ,  $\frac{9}{10}$ ,  $\frac{23}{25}$ 

Numerator is less than denominator. Value is less than 1.

Improper fractions  $\frac{3}{2}$ ,  $\frac{5}{5}$ ,  $\frac{11}{7}$ ,

Numerator is greater than or equal to denominator. Value is greater than or equal to 1.

#### **OBJECTIVE 1** Write fractions in lowest terms.

A fraction is in **lowest terms** when the numerator and denominator have no factors in common (other than 1).

#### Writing a Fraction in Lowest Terms

- Step 1 Write the numerator and denominator in factored form.
- **Step 2** Replace each pair of factors common to the numerator and denominator with 1.
- **Step 3** Multiply the remaining factors in the numerator and in the denominator.

(This procedure is sometimes called "simplifying the fraction.")

#### **VOCABULARY**

☐ fractions

2

- □ numerator
- □ denominator
- □ proper fraction
- ☐ improper fraction
- □ lowest terms
- ☐ mixed number
- □ reciprocals
- □ decimal



Write each fraction in lowest

(a) 
$$\frac{30}{42}$$

**(b)** 
$$\frac{10}{70}$$

NOW TRY EXERCISE 2

Write  $\frac{92}{5}$  as a mixed number.

**o**) 
$$\frac{10}{70}$$
 (c)

#### **EXAMPLE 1** Writing Fractions in Lowest Terms

Write each fraction in lowest terms.

(a) 
$$\frac{10}{15} = \frac{2 \cdot 5}{3 \cdot 5} = \frac{2}{3} \cdot \frac{5}{5} = \frac{2}{3} \cdot 1 = \frac{2}{3}$$
 5 is the greatest common factor of 10 and 15.

**(b)** 
$$\frac{15}{45} = \frac{15}{3 \cdot 15} = \frac{1}{3 \cdot 1} = \frac{1}{3}$$
 Remember to write 1 in the numerator.

(c) 
$$\frac{150}{200} = \frac{3 \cdot 50}{4 \cdot 50} = \frac{3}{4} \cdot 1 = \frac{3}{4}$$
 50 is the greatest common factor of 150 and 200.

Another strategy is to choose any common factor and work in stages.

$$\frac{150}{200} = \frac{15 \cdot 10}{20 \cdot 10} = \frac{3 \cdot 5 \cdot 10}{4 \cdot 5 \cdot 10} = \frac{3}{4} \cdot 1 \cdot 1 = \frac{3}{4}$$
 The same answer results.





#### **OBJECTIVE 2** Convert between improper fractions and mixed numbers.

A mixed number is a single number that represents the sum of a natural (counting) number and a proper fraction.

Mixed number 
$$\rightarrow 2\frac{3}{4} = 2 + \frac{3}{4}$$

#### **EXAMPLE 2** Converting an Improper Fraction to a Mixed Number

Write  $\frac{59}{8}$  as a mixed number.

Because the fraction bar represents division  $\left(\frac{a}{b} = a \div b, \text{ or } b\right)\overline{a}$ , divide the numerator of the improper fraction by the denominator.

Denominator of fraction 
$$\rightarrow 8)\overline{59}$$
  $\leftarrow$  Quotient  $\overline{56}$  Numerator of fraction  $\overline{59}$  =  $7\frac{3}{8}$  Now try

# NOW TRY EXERCISE 3

Write  $11\frac{2}{3}$  as an improper

#### **EXAMPLE 3** Converting a Mixed Number to an Improper Fraction

Write  $6\frac{4}{7}$  as an improper fraction.

Multiply the denominator of the fraction by the natural number, and then add the numerator to obtain the numerator of the improper fraction

$$7 \cdot 6 = 42$$
 and  $42 + 4 = 46$ 

The denominator of the improper fraction is the same as the denominator in the mixed number, which is 7 here.

$$6\frac{4}{7} = \frac{7 \cdot 6 + 4}{7} = \frac{46}{7}$$



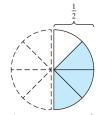


- □ terminating decimal
- ☐ repeating decimal
- □ percent

- (a)  $\frac{30}{42}$  (b)  $\frac{10}{70}$  (c)  $\frac{72}{120}$

- **NOW TRY ANSWERS** 1. (a)  $\frac{5}{7}$  (b)  $\frac{1}{7}$  (c)  $\frac{3}{5}$
- 2.  $18\frac{2}{5}$

NOW TRY



 $\frac{3}{4}$  of  $\frac{1}{2}$  is equivalent to  $\frac{3}{4} \cdot \frac{1}{2}$ , which equals  $\frac{3}{8}$  of the circle.

FIGURE 2



Multiply. Write the answer in lowest terms.

$$\frac{4}{7} \cdot \frac{5}{8}$$

#### **OBJECTIVE 3** Perform operations with fractions.

FIGURE 2 illustrates multiplying fractions.

#### Multiplying Fractions

If 
$$\frac{a}{b}$$
 and  $\frac{c}{d}$  are fractions, then  $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$ .

That is, to multiply two fractions, multiply their numerators and then multiply their denominators.

#### **EXAMPLE 4** Multiplying Fractions

Multiply. Write the answer in lowest terms.

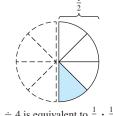
$$\frac{3}{8} \cdot \frac{4}{9}$$

$$= \frac{3 \cdot 4}{8 \cdot 9}$$
Multiply numerators.
$$= \frac{12}{72}$$
Multiply.
$$= \frac{1 \cdot 12}{6 \cdot 12}$$
The greatest common factor of 12 and 72 is 12.

Make sure the product is in lowest terms.
$$= \frac{1}{6}$$

$$\frac{1 \cdot 12}{6 \cdot 12} = \frac{1}{6} \cdot 1 = \frac{1}{6}$$

Two numbers are **reciprocals** of each other if their product is 1. For example,  $\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12}$ , or 1. Division is the inverse or opposite of multiplication, and as a result, we use reciprocals to divide fractions. **FIGURE 3** illustrates dividing fractions.



 $\frac{1}{2} \div 4$  is equivalent to  $\frac{1}{2} \cdot \frac{1}{4}$ , which equals  $\frac{1}{8}$  of the circle.

FIGURE 3

#### **Dividing Fractions**

If 
$$\frac{a}{b}$$
 and  $\frac{c}{d}$  are fractions, then  $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$ .

That is, to divide by a fraction, multiply by its reciprocal.

#### **EXAMPLE 5** Dividing Fractions

Divide. Write answers in lowest terms as needed.

(a) 
$$\frac{3}{4} \div \frac{8}{5}$$
 (b)  $\frac{5}{8} \div 10$  Think of  $10$  as  $\frac{10}{1}$  here.

$$= \frac{3}{4} \cdot \frac{5}{8} \quad \text{Multiply by } \frac{5}{8}, \text{ the reciprocal of } \frac{8}{5}.$$

$$= \frac{3 \cdot 5}{4 \cdot 8} \quad \text{Multiply numerators.}$$

$$= \frac{15}{32} \quad \text{Make sure the answer is in lowest terms.}$$
(b)  $\frac{5}{8} \div 10$  Think of  $10$  as  $\frac{10}{1}$  here.

$$= \frac{5}{8} \cdot \frac{1}{10} \quad \text{Multiply by } \frac{1}{10}, \text{ the reciprocal of } 10.$$

$$= \frac{5 \cdot 1}{8 \cdot 2 \cdot 5} \quad \text{Multiply and factor.}$$

$$= \frac{1}{16} \quad \text{Remember to write } 1 \quad \text{in the numerator.}$$

#### **NOW TRY ANSWER**

4.  $\frac{5}{14}$ 



Divide. Write answers in lowest terms as needed.

(a) 
$$\frac{2}{7} \div \frac{8}{9}$$

4

(a) 
$$\frac{2}{7} \div \frac{8}{9}$$
 (b)  $3\frac{3}{4} \div 4\frac{2}{7}$ 

(c)  $1\frac{2}{3} \div 4\frac{1}{2}$ 

$$=\frac{5}{3} \div \frac{9}{2}$$

Write each mixed number as an improper fraction.

$$=\frac{5}{3}\cdot\frac{2}{9}$$

 $= \frac{5}{3} \cdot \frac{2}{9}$  Multiply by  $\frac{2}{9}$ , the reciprocal of  $\frac{9}{2}$ .

$$=\frac{10}{27}$$

Multiply. The quotient is in lowest terms.





FIGURES 4 and 5 illustrate adding and subtracting fractions.

#### **Adding Fractions**

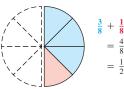
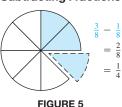


FIGURE 4

#### **Subtracting Fractions**



#### Adding and Subtracting Fractions

If  $\frac{a}{b}$  and  $\frac{c}{b}$  are fractions (where  $b \neq 0$ ), then add or subtract as follows.

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

$$\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$$

That is, to add or subtract two fractions having the same denominator, add or subtract the numerators and keep the same denominator.

If the denominators are different, first find the least common denominator (LCD). Write each fraction as an equivalent fraction with this denominator. Then add or subtract as above.

#### **EXAMPLE 6** Adding and Subtracting Fractions

Add or subtract as indicated. Write answers in lowest terms as needed.

(a) 
$$\frac{2}{10} + \frac{3}{10}$$

**(b)** 
$$\frac{4}{15} + \frac{5}{9}$$

$$=\frac{2+3}{10}$$

$$= \frac{2+3}{10}$$
 Add numerators. Keep the same 
$$= \frac{4}{15} \cdot \frac{3}{3} + \frac{5}{9} \cdot \frac{5}{5}$$
 Add numerators. LCD is  $3 \cdot 3 \cdot 5$ , so the LCD is  $3 \cdot 3 \cdot 5 = 45$ .

$$=\frac{5}{10}$$

$$=\frac{12}{45}+\frac{25}{45}$$

Write equivalent common denominator.

$$=\frac{1}{2}$$

Write in lowest

$$=\frac{37}{45}$$

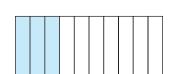
Add numerators. Keep the same denominator.

**NOW TRY ANSWERS** 5. (a)  $\frac{9}{28}$  (b)  $\frac{7}{8}$ 



Add or subtract as indicated. Write answers in lowest terms as needed.

- (a)  $\frac{1}{8} + \frac{3}{8}$  (b)  $\frac{5}{12} + \frac{3}{8}$
- (c)  $\frac{5}{11} \frac{2}{9}$  (d)  $4\frac{1}{3} 2\frac{5}{6}$



3 parts of the whole 10 are shaded. As a fraction,  $\frac{3}{10}$  of the figure is shaded. As a decimal, 0.3 is shaded. Both of these numbers are read "three-tenths."

#### FIGURE 6

(c) 
$$\frac{15}{16} - \frac{4}{9}$$

$$= \frac{15}{16} \cdot \frac{9}{9} - \frac{4}{9} \cdot \frac{16}{16}$$

$$= \frac{135}{144} - \frac{64}{144}$$

Because 16 and 9 have no common factors except 1, the LCD is

Write equivalent fractions with the common denominator.

Subtract numerators.

Keep the common denominator.

**(d)** 
$$4\frac{1}{2} - 1\frac{3}{4}$$

Method 1 
$$4\frac{1}{2} - 1\frac{3}{4}$$

$$= \frac{9}{2} - \frac{3}{4}$$

Write each mixed number as an improper fraction.

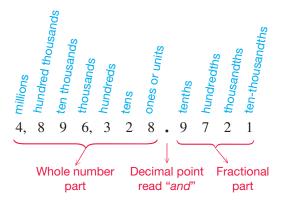
$$= \frac{18}{4} - \frac{7}{4}$$
 Find a common denominator. The LCD is 4. 
$$= \frac{11}{4}, \quad \text{or} \quad 2\frac{3}{4}$$
 Subtract. Write as a mixed number.

Method 2 
$$4\frac{1}{2} = 4\frac{2}{4} = 3\frac{6}{4}$$
 The LCD is 4. 
$$4\frac{2}{4} = 3 + 1 + \frac{2}{4} = 3 + \frac{4}{4} + \frac{2}{4} = 3\frac{6}{4}$$
 
$$-1\frac{3}{4} = 1\frac{3}{4} = 1\frac{3}{4}$$
 
$$2\frac{3}{4}$$
, or  $\frac{11}{4}$  The same answer results.



Fractions are one way to represent parts of a whole. Another way is with a decimal fraction or **decimal**, a number written with a decimal point.

See **FIGURE 6**. Each digit in a decimal number has a place value, as shown below.



Each successive place value is ten times greater than the place value to its right and one-tenth as great as the place value to its left.

#### **NOW TRY ANSWERS**

**6.** (a)  $\frac{1}{2}$  (b)  $\frac{19}{24}$  (c)  $\frac{23}{99}$ (d)  $\frac{3}{2}$ , or  $1\frac{1}{2}$ 

#### **OBJECTIVE 4** Write decimals as fractions.

Place value is used to write a decimal number as a fraction.

#### Converting a Decimal to a Fraction

Read the decimal using the correct place value. Write it in fractional form just as it is read.

- The numerator will be the digits to the right of the decimal point.
- The denominator will be a power of 10—that is, 10 for tenths, 100 for hundredths, and so on.

#### **EXAMPLE 7** Writing Decimals as Fractions

Write each decimal as a fraction. (Do not write in lowest terms.)

We read 0.95 as "ninety-five hundredths." (a) 0.95

$$0.95 = \frac{95}{100} \leftarrow \text{For hundredths}$$

We read 0.056 as "fifty-six thousandths." **(b)** 0.056

Do not confuse **0.056** with **0.56**, read "fifty-six *hundredths*," which is the fraction 
$$\frac{56}{100}$$
.  $0.056 = \frac{56}{1000}$  — For thousandths

(c) 4.2095 We read this decimal number, which is greater than 1, as "Four and two thousand ninety-five ten-thousandths."

$$4.2095 = 4\frac{2095}{10,000}$$
 Write the decimal number as a mixed number.

Think: 
$$10,000 \cdot 4 + 2095 = 42,095 \over 10,000$$

Write the mixed number as an improper fraction.



#### **OBJECTIVE 5** Perform operations with decimals.

# **EXAMPLE 8** Adding and Subtracting Decimals

Add or subtract as indicated.

(a) 
$$6.92 + 14.8 + 3.217$$

Place the digits of the decimal numbers in columns by place value. Attach zeros as placeholders so that there are the same number of places to the right of each decimal point.

(b) 
$$47.6 - 32.509$$

$$47.6 - 32.509$$
 becomes  $47.600$  Write the decimal numbers in columns, attaching 0s to 47.6.

# **NOW TRY**

6

Write each decimal as a fraction. (Do not write in lowest terms.)

(a) 0.8 (b) 0.431 (c) 2.58

Add or subtract as indicated.

(a) 
$$68.9 + 42.72 + 8.973$$

**(b)** 
$$351.8 - 2.706$$

#### **NOW TRY ANSWERS**

- 7. (a)  $\frac{8}{10}$  (b)  $\frac{431}{1000}$  (c)  $\frac{258}{100}$
- **8.** (a) 120.593 (b) 349.094



Multiply or divide as indicated.

- (a)  $9.32 \times 1.4$
- **(b)**  $0.6 \times 0.004$
- (c) 7.334 ÷ 3.1 (Round the answer to two decimal places.)

#### **EXAMPLE 9** Multiplying and Dividing Decimals

Multiply or divide as indicated.

(a)  $29.3 \times 4.52$ 

Multiply normally. Place the decimal point in the answer as shown.

**(b)**  $0.05 \times 0.3$ 

Here  $5 \times 3 = 15$ . Be careful placing the decimal point.

2 decimal places 1 decimal place

$$0.05 \times 0.3$$
Do not write 0.150. 
$$= 0.015$$

2 + 1 = 3 decimal places Attach 0 as a placeholder in the tenths place.

(c)  $8.949 \div 1.25$  (Round the answer to two decimal places.)

We carried out the division to three decimal places so that we could round to two decimal places, obtaining the answer 7.16.

**NOTE** To round 7.159 in **Example 9(c)** to two decimal places (that is, to the nearest hundredth), we look at the digit to the *right* of the hundredths place.

- If this digit is 5 or greater, we round up.
- If this digit is less than 5, we drop the digit(s) beyond the desired place.

Hundredths place

7.159

9, the digit to the right of the hundredths place, is 5 or greater.

≈ 7.1<mark>6</mark>

Round 5 up to 6.  $\approx$  means is approximately equal to.

#### Multiplying and Dividing by Powers of 10 (Shortcuts)

- To *multiply* by a power of 10, *move the decimal point to the right* as many places as the number of zeros.
- To *divide* by a power of 10, *move the decimal point to the left* as many places as the number of zeros.

In both cases, insert 0s as placeholders if necessary.

#### **NOW TRY ANSWERS**

**9. (a)** 13.048 **(b)** 0.0024

**(c)** 5.64

8

- (a)  $294.72 \times 10$
- **(b)** 4.793 ÷ 100

#### EXAMPLE 10 Multiplying and Dividing by Powers of 10

Multiply or divide as indicated.

(a)  $48.731 \times 10$ 

=	48.731
=	487.31

Move the decimal point one place to the *right*.

= 48.731

**(b)**  $48.731 \times 1000$ 

Move the decimal point three places to the *right*.

(c)  $48.731 \div 10$ 

$$= 48.731$$
  
= 4.8731

Move the decimal point one place to the *left*.

(d)  $48.731 \div 1000$ 

$$= 048.731$$
  
= 0.048731

= 48.731

Move the decimal point three places to the *left*.



#### NUW

#### **OBJECTIVE 6** Write fractions as decimals.

#### Converting a Fraction to a Decimal

Because a fraction bar indicates division, write a fraction as a decimal by dividing the numerator by the denominator.

#### NOW TRY EXERCISE 11

Write each fraction as a decimal. For repeating decimals, write the answer by first using bar notation and then rounding to the nearest thousandth.

(a)  $\frac{17}{20}$  (b)  $\frac{2}{9}$ 

#### **EXAMPLE 11** Writing Fractions as Decimals

Write each fraction as a decimal.

(a)  $\frac{19}{8}$ 

$$\begin{array}{r}
2.375 \\
8) 19.000 \\
\underline{16} \\
30 \\
\underline{24} \\
60 \\
\underline{56} \\
40 \\
40
\end{array}$$

Divide 19 by 8. Add a decimal point and as many 0s as necessary to 19.

 $\begin{array}{c} \textbf{(b)} \ \frac{2}{3} & \frac{0.6666 \dots}{3)2.0000 \dots} \\ & \frac{18}{20} \\ & \frac{18}{20} \\ & \frac{18}{20} \\ & \frac{18}{20} \end{array}$ 

$$\frac{19}{8} = 2.375 \leftarrow \text{Terminating decimal}$$

$$\frac{2}{3} = 0.6666 \dots$$
 Repeating decimal
$$= 0.\overline{6}$$
 A bar is written over the repeating digit(s).
$$\approx 0.667$$
 Nearest thousandth

NOW TRY



#### **OBJECTIVE 7** Write percents as decimals and decimals as percents.

The word **percent** means "per 100." Percent is written with the symbol %. "One percent" means "one per one hundred," or "one one-hundredth." See FIGURE 7.

#### **NOW TRY ANSWERS**

**10. (a)** 2947.2 **(b)** 0.04793

35 of the 100 squares are

of the figure is shaded. **FIGURE 7** 

shaded. That is,  $\frac{35}{100}$ , or 35%,

**11.** (a) 0.85 (b)  $0.\overline{2}$ ; 0.222

#### Percent, Fraction, and Decimal Equivalents

$$1\% = \frac{1}{100} = 0.01, \quad 10\% = \frac{10}{100} = 0.10, \quad 100\% = \frac{100}{100} = 1$$

For example, 73% means "73 per one hundred."

$$73\% = \frac{73}{100} = 0.73$$

Essentially, we are dropping the % symbol from 73% and dividing 73 by 100. Doing this moves the decimal point, which is understood to be after the 3, two places to the left.

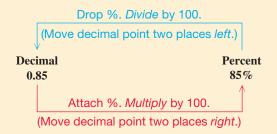
Writing 0.73 as a percent is the opposite process.

$$0.73 = 0.73 \cdot 100\% = 73\%$$
  $100\% = 1$ 

Moving the decimal point two places to the *right* and attaching a % symbol give the same result.

#### **Converting Percents and Decimals (Shortcuts)**

- To convert a percent to a decimal, move the decimal point two places to the *left* and drop the % symbol.
- To convert a decimal to a percent, move the decimal point two places to the right and attach a % symbol.



#### **NOW TRY EXERCISE 12**

Convert each percent to a decimal and each decimal to a percent.

- (a) 52% (b) 2%
- (c) 0.45 (d) 3.5

#### EXAMPLE 12 Converting Percents and Decimals by Moving the Decimal Point

Convert each percent to a decimal and each decimal to a percent. Convert each percent to a decimal and (a) 45% = 0.45 (b) 250% = 2.50, or 2.5 (c) 9% = 0.09 (d) 0.57 = 57% (e) 1.5 = 1.50 = 150% (f) 0.007 = 0.7%

#### **OBJECTIVE 8** Write percents as fractions and fractions as percents.

#### **EXAMPLE 13** Writing Percents as Fractions

Write each percent as a fraction. Give answers in lowest terms.

(a) 8%

We use the fact that *percent* means "per one hundred," and convert as follows.

$$8\% = \frac{8}{100} = \frac{2 \cdot 4}{25 \cdot 4} = \frac{2}{25}$$
 As with converting percents to decimals, drop the % symbol and divide by 100.

**(b)** 
$$175\% = \frac{175}{100} = \frac{7 \cdot 25}{4 \cdot 25} = \frac{7}{4}$$
, or  $1\frac{3}{4}$  A number greater than 1 is more than 100%.

Write in lowest terms.